EUROPEAN JOURNAL OF PURE AND APPLIED MATHEMATICS
2025, Vol. 18, Issue 2, Article Number 5980
ISSN 1307-5543 – ejpam.com
Published by New York Business Global

Bio-Inspired Optimization Through Photosynthesis: A Novel Approach for Balancing Exploration and Exploitation in Complex Systems

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Abstract. This paper presents the Photosynthesis-Inspired Optimization (PSIO) algorithm, an innovative metaheuristic that emulates the adaptive efficiency of plant photosynthesis. Inspired by chlorophyll's dual role in light absorption and energy conversion, PSIO operates through two core mechanisms: the Photo-Intensification Mechanism, which focuses on refining high-potential solutions, and the Photosynthetic Pathway Diversification, which promotes thorough exploration of the solution space. These interconnected strategies enable PSIO to maintain a dynamic balance between exploration and exploitation, reducing the likelihood of premature convergence - a common challenge in complex optimization problems. Additionally, the algorithm incorporates an adaptive adjustment mechanism, akin to the photoprotective responses in plants, enhancing its flexibility and robustness across various optimization scenarios. The effectiveness of PSIO is validated through extensive benchmarking, consistently outperforming conventional metaheuristics in both convergence speed and solution quality. More specifically, PSIO consistently achieves lower total transmission costs with improvements ranging from 10% to 25% for small and large-scale problems.

2020 Mathematics Subject Classifications: 8T20, 90C59, 68W10, 90C26

Key Words and Phrases: Artificial Intelligence, Metaheuristic, Optimization Algorithms, Adaptive Adjustment

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DOI: https://doi.org/10.29020/nybg.ejpam.v18i2.5980

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1. Introduction

Optimization is a fundamental element across various disciplines, including engineering, logistics, finance, and machine learning. Within these fields, the pursuit of optimal solutions from an extensive set of possibilities is pivotal for enhancing efficiency, minimizing costs, and improving overall system performance. While traditional optimization techniques, such as gradient descent and linear programming, have demonstrated effectiveness in certain contexts, they frequently encounter significant challenges when faced with the complexity and high dimensionality characteristic of modern problems. These challenges often result in convergence to local optima, limiting the effectiveness of deterministic approaches.

To address these limitations, nature-inspired metaheuristics have emerged as powerful and flexible tools capable of navigating complex optimization landscapes. These algorithms are inspired by a wide array of natural processes, including biological evolution, swarm intelligence, and neural activities, which offer robust strategies for exploration and exploitation. By emulating these processes, metaheuristics extend the reach and effectiveness of traditional optimization techniques.

Photosynthesis, in particular, presents a striking example of an efficient natural process. Chlorophyll, the pigment responsible for light absorption in plants, demonstrates adaptive behaviors that are highly relevant to optimization. The precision with which chlorophyll optimizes light absorption and energy conversion can be analogized to the balance between exploration and exploitation in optimization algorithms-a balance that is crucial for avoiding local optima and achieving global optimization.

Despite the advancements made by existing metaheuristic algorithms, challenges such as premature convergence and difficulties in handling multimodal or deceptive landscapes persist. These issues often arise from an imbalance between intensification—where the focus is on refining promising solutions—and diversification, which involves exploring new regions of the solution space. Achieving effective optimization necessitates a dynamic equilibrium between these two strategies, ensuring comprehensive exploration without neglecting the enhancement of identified solutions.

The adaptive strategies exhibited by chlorophyll in response to environmental variations provide a compelling model for addressing these challenges. Chlorophyll's ability to finely tune its light absorption and energy conversion processes in response to changing conditions mirrors the adaptive balance required in optimization algorithms. By drawing inspiration from these natural mechanisms, we propose a novel approach to optimization that emulates the dynamic and efficient strategies of photosynthesis, offering a promising direction for overcoming the limitations of current metaheuristics.

1.1. Background and Motivation

Optimization plays a critical role in various fields, including engineering, logistics, finance, and machine learning, where the goal is to identify the optimal solution from a large set of possibilities [1, 2]. This process is essential for enhancing efficiency, reducing costs, and improving overall performance in numerous applications. Traditional optimization methods, such as gradient descent and linear programming, are effective in certain contexts but often struggle with complex or high-dimensional problems due to their deterministic nature and tendency to converge to local optima [3].

In recent years, nature-inspired metaheuristics have gained prominence as versatile tools capable of addressing these challenges. These algorithms draw inspiration from biological, physical, and ecological systems, leveraging natural processes such as evolution, swarm intelligence, and neural activities to explore complex solution landscapes [4, 5]. A particularly intriguing source of inspiration is photosynthesis, the process by which plants convert solar energy into chemical energy. Chlorophyll, a vital component in this process, demonstrates adaptive behaviors that parallel optimization strategies, such as efficiently capturing light and converting it into energy, which can be likened to balancing exploration and exploitation in optimization [6].

1.2. Challenges in Current Metaheuristics

Despite the successes of metaheuristic algorithms, they face significant challenges, including premature convergence and entrapment in local optima, particularly in multimodal or deceptive landscapes [7, 8]. These issues often result from an imbalance between intensification (focusing on exploiting known good solutions) and diversification (exploring new areas of the solution space) [9]. Effective optimization requires a dynamic balance between these strategies to avoid premature convergence and ensure a thorough exploration of the solution space.

The adaptive nature of chlorophyll under varying environmental conditions offers a promising model for developing new optimization approaches. Chlorophyll's dual strategy of light absorption and energy conversion can be conceptually linked to the balance of intensification and diversification needed in optimization algorithms [6].

1.3. Objective of the Study

This research introduces the Photosynthesis-Inspired Optimization (PSIO) algorithm, a novel metaheuristic inspired by the adaptive mechanisms of chlorophyll in photosynthesis. The algorithm enhances the optimization process by emulating chlorophyll's dual strategy: light absorption represents intensification, while energy conversion facilitates diversification [10]. By integrating these principles, PSIO aims to achieve an optimal balance between exploration and exploitation, enabling more effective navigation of complex and high-dimensional search spaces.

Key contributions of this study include the development of the PSIO framework, a comparative evaluation against existing metaheuristic algorithms, and empirical validation through benchmark optimization problems. Ultimately, this research seeks to advance metaheuristic optimization by offering a robust and adaptable approach applicable to a wide range of complex optimization tasks [4].

1.4. Research Questions and Hypothesis

This study is guided by the following research questions and hypotheses:

- *RQ1*: How does the PSIO algorithm compare to traditional metaheuristic algorithms in terms of convergence speed and solution quality?
- *Hypothesis 1:* The PSIO algorithm will demonstrate superior convergence speed and solution quality compared to traditional metaheuristic algorithms due to its balanced intensification and diversification strategies.
- *RQ2*: Can the adaptive adjustment mechanism within the PSIO algorithm effectively prevent premature convergence in complex optimization landscapes?
- *Hypothesis 2:* The adaptive adjustment mechanism will effectively balance exploration and exploitation, thereby preventing premature convergence and improving solution robustness.
- *RQ3*: What impact does the PSIO algorithm have on solving high-dimensional and multimodal optimization problems?
- *Hypothesis 3:* The PSIO algorithm will be particularly effective in high-dimensional and multimodal optimization problems, demonstrating improved performance over existing algorithms.

1.5. Research Contribution

This paper presents several key contributions to the field of metaheuristic optimization through the development and validation of the Photosynthesis-Inspired Optimization (PSIO) algorithm:

(i) **Novel Metaheuristic Algorithm:** The introduction of the PSIO algorithm, which emulates the adaptive strategies of chlorophyll during photosynthesis, provides a unique approach to balancing exploration and exploitation in optimization problems. This dual strategy, inspired by natural photosynthetic processes, enhances the algorithm's capability to navigate complex solution landscapes effectively.

(ii) **Adaptive Adjustment Mechanism:** The development of an adaptive adjustment mechanism within the PSIO algorithm, which dynamically balances intensification and diversification based on convergence rate and population diversity, contributes to avoiding premature convergence and improving solution quality.

(iii) **Comparative Analysis:** A comprehensive comparative analysis of the PSIO algorithm against existing metaheuristic algorithms, demonstrating superior performance in terms of convergence speed and solution quality on benchmark problems. This analysis highlights the efficacy of PSIO in handling multimodal and deceptive optimization landscapes.

(iv) **Empirical Validation:** Empirical validation of the PSIO algorithm through its application to benchmark optimization problems, specifically the capacitated single-allocation p-hub location-routing problem. The results showcase the PSIO algorithm's ability to achieve lower total costs and comparable computation times relative to traditional hyper-heuristic approaches.

(v) **Framework for Future Research:** The conceptual framework and implementation details provided for the PSIO algorithm lay the groundwork for future research. This includes exploring variations of the PSIO algorithm, integrating it with other optimization techniques, and applying it to a broader range of complex real-world problems.

(vi) **Theoretical Insights:** The paper contributes to the theoretical understanding of nature-inspired optimization algorithms by elucidating the parallels between photosynthetic processes and optimization strategies. This interdisciplinary approach opens new avenues for developing more robust and versatile optimization algorithms.

These contributions collectively advance the field of metaheuristic optimization, offering both theoretical insights and practical solutions to complex optimization challenges.

1.6. Paper Overview

This paper is organized into several sections to provide a structured exploration of the Photosynthesis-Inspired Optimization (PSIO) algorithm. Section 2 presents the theoretical background, discussing the biological inspiration behind PSIO, particularly the role of chlorophyll in optimizing light absorption and energy conversion. It also reviews fundamental metaheuristic optimization principles and related nature-inspired algorithms. Section 3 introduces the PSIO algorithm, detailing its conceptual framework and key components, including Chlorophyll-Like Molecules (CLMs), the Photo-Intensification Mechanism, the Photosynthetic Pathway Diversification process, and the Adaptive Adjustment Mechanism. It further discusses PSIO's applicability in both single-based and population-based optimization contexts. Section 4 focuses on the implementation of the algorithm, outlining its structure, parameter settings, and computational complexity. This section highlights how the balance between exploration and exploitation is dynamically maintained throughout the optimization process. Section 5 presents experimental results and performance evaluation. The PSIO algorithm is tested against benchmark problems and compared with existing metaheuristics based on convergence speed, solution quality, and computational efficiency. The results demonstrate the algorithm's effectiveness in solving complex optimization problems. Finally, Section 6 concludes the paper by summarizing key contributions and suggesting future research directions. The potential for hybridizing PSIO with other optimization techniques and extending its application to real-world problems is emphasized as an avenue for further investigation.

2. Theoretical Background

2.1. Photosynthesis in Nature

Photosynthesis is a complex biochemical process that is fundamental to life on Earth, enabling plants, algae, and certain bacteria to convert light energy into chemical energy in the form of glucose while releasing oxygen as a byproduct [11, 12]. This process, occurring within the chloroplasts of plant cells, plays a crucial role in the global carbon cycle and serves as the primary energy source for nearly all living organisms [13, 14].

Central to photosynthesis is chlorophyll, the green pigment that absorbs light, particularly in the blue and red wavelengths, and transfers this energy to drive the chemical reactions that produce glucose from carbon dioxide and water [15–17]. The efficiency of photosynthesis is not static; it varies with several environmental factors, such as light intensity, wavelength, temperature, and the availability of water and nutrients [18–20]. Plants have evolved sophisticated adaptive mechanisms to optimize energy capture even under fluctuating environmental conditions [21–23]. For instance, under low light conditions, plants increase chlorophyll production to maximize light absorption, whereas, in high light conditions, protective mechanisms are activated to prevent photodamage [20, 21].

The ability of chlorophyll to adjust its function according to the surrounding environment is analogous to the concept of balancing intensification and diversification in optimization algorithms [24, 25]. Intensification involves focusing on exploiting known good solutions to further refine them, much like chlorophyll optimizing light absorption under favorable conditions [23]. Diversification, on the other hand, entails exploring new areas of the solution space to avoid local optima and discover potentially better solutions, similar to how plants explore alternative photosynthetic pathways under less-than-ideal conditions [26, 27].

This dynamic adaptation ensures that plants can maximize their energy absorption and overall efficiency by striking a balance between exploiting optimal conditions and adapting to less favorable ones. The principles of photosynthesis, particularly the strategies employed by chlorophyll, offer valuable insights into developing robust optimization algorithms [20, 25]. By mimicking these natural processes, optimization algorithms such as Photosynthesis-Inspired Optimization (PSIO) can significantly improve their ability to navigate complex solution landscapes and achieve high-quality solutions [24, 26].

2.2. Metaheuristic Optimization Principles

Metaheuristic optimization refers to a broad class of algorithms designed to find approximate solutions to complex optimization problems that may be difficult or impossible to solve exactly due to their size or complexity [28, 29]. These algorithms leverage heuristic strategies to explore and exploit the solution space, making them particularly useful for problems characterized by large search spaces or unknown solution structures [30, 31]. Metaheuristics have been successfully applied across various domains, including engineering, logistics, finance, and machine learning, demonstrating their versatility and robustness in handling diverse optimization tasks [32, 33].

A critical aspect of metaheuristic algorithms is their ability to balance two fundamental principles: intensification and diversification. Intensification focuses on exploiting the most promising solutions discovered during the search process, refining them to achieve better performance [32, 33]. This approach is analogous to chlorophyll's focused absorption of light under optimal conditions, where the plant maximizes energy capture and conversion efficiency [23]. For example, in Particle Swarm Optimization (PSO), particles adjust their trajectories based on the best solutions found by themselves and their neighbors, thus intensifying the search around these promising areas [34, 35].

Diversification, on the other hand, involves exploring new areas of the search space to avoid local optima and ensure a comprehensive search [32, 36]. This can be compared to the plant's ability to utilize alternative photosynthetic pathways when light conditions are not ideal, thereby maintaining overall photosynthetic efficiency [21, 22]. In Genetic Algorithms (GA), for instance, mutation and crossover operations introduce genetic diversity into the population, enabling the exploration of new regions in the solution space [29, 37].

Several well-known metaheuristic algorithms incorporate these principles in different ways. Simulated Annealing (SA), for example, mimics the annealing process in metallurgy, where a material is heated and then slowly cooled to remove defects, representing a balance between exploration (high temperature) and exploitation (low temperature) [30, 38]. Similarly, Ant Colony Optimization (ACO) is inspired by the foraging behavior of ants, where pheromone trails guide the search process, balancing between intensification (following strong pheromone trails) and diversification (exploring new paths) [36, 39].

Moreover, hybrid metaheuristics combine the strengths of multiple algorithms to enhance performance further. For instance, Hybrid Genetic Algorithms (HGA) integrate the evolutionary strategies of GA with local search methods to improve both the exploration and exploitation capabilities [29, 33]. This hybridization often leads to more efficient and effective optimization solutions, especially in complex and high-dimensional problem spaces [30, 32].

The dynamic balance between intensification and diversification is crucial for the success of metaheuristic algorithms. Excessive intensification can lead to premature convergence, where the algorithm gets stuck in local optima, while too much diversification may result in slow convergence and inefficient search processes [24, 32]. Adaptive mechanisms are often employed to dynamically adjust this balance based on the progress of the search, enhancing the algorithm's ability to find high-quality solutions in a reasonable timeframe [26, 31].

The PSIO algorithm, as proposed in this study, leverages these principles in a novel way. By emulating the adaptive strategies of chlorophyll in photosynthesis, PSIO introduces Chlorophyll-Like Molecules (CLMs) that represent candidate solutions, a Photo-Intensification Mechanism for local exploitation, and a Photosynthetic Pathway Diversification process for global exploration [26, 27]. This biologically-inspired approach aims to achieve a robust and flexible optimization performance, drawing from the inherent efficiency of natural processes [32, 33].

2.3. Related Work

The field of nature-inspired optimization has seen significant developments over the past few decades, with various algorithms being proposed and refined to tackle complex optimization problems. These algorithms draw inspiration from a wide range of natural processes, including biological evolution, swarm intelligence, and social behaviors, each contributing unique strategies for exploration and exploitation within the solution space.

Genetic Algorithms (GAs) are among the earliest and most well-known nature-inspired optimization techniques. Introduced by John Holland in the 1970s, GAs are based on the principles of natural selection and genetics [37]. They have been widely applied to optimization problems across various domains, including engineering, economics, and artificial intelligence [40, 41]. Despite their popularity, GAs often struggle with maintaining diversity in the population as the search progresses, which can lead to premature convergence to suboptimal solutions [42]. Several enhancements, such as elitism, adaptive mutation rates, and hybridization with local search methods, have been proposed to address these challenges [43, 44].

Particle Swarm Optimization (PSO), introduced by Kennedy and Eberhart in 1995, is another widely used nature-inspired algorithm that mimics the social behavior of birds flocking or fish schooling [34]. PSO has gained popularity due to its simplicity and effectiveness in solving a variety of optimization problems, including those in engineering design, neural network training, and economic modeling [45, 46]. However, PSO is not without its limitations; it can suffer from premature convergence, particularly in high-dimensional or complex landscapes where particles may cluster around local optima [47, 48]. To mitigate these issues, various modifications, such as the introduction of inertia weight, constriction factors, and multi-swarm approaches, have been proposed [49, 50].

Ant Colony Optimization (ACO), inspired by the foraging behavior of ants, was developed by Dorigo and colleagues in the early 1990s [36]. ACO has proven to be particularly effective in solving combinatorial optimization problems, such as the traveling salesman problem (TSP), vehicle routing problems (VRP), and scheduling tasks [51–53]. ACO's ability to dynamically balance exploration and exploitation through the use of pheromone trails has been a key factor in its success [54]. However, similar to other nature-inspired algorithms, ACO can be prone to premature convergence, especially when the pheromone trails become too strong, leading to a lack of diversity in the search [39, 55]. Various strategies, such as pheromone evaporation, hybridization with other metaheuristics, and the use of multi-colony systems, have been explored to address these limitations [56, 57].

Simulated Annealing (SA), another cornerstone of metaheuristic optimization, draws inspiration from the annealing process in metallurgy [38]. SA has been successfully applied to a wide range of optimization problems, particularly those involving large and complex search spaces [58, 59]. The strength of SA lies in its ability to escape local optima by allowing occasional uphill moves, thus facilitating a more comprehensive exploration of the solution space [60, 61]. Despite its robustness, SA can be computationally expensive, especially when the cooling schedule is slow, and finding an optimal balance between exploration and exploitation remains a challenge [62, 63].

Hybrid metaheuristics have gained increasing attention as researchers seek to combine the strengths of different algorithms to enhance their performance [64, 65]. For instance, Hybrid Genetic Algorithms (HGA) integrate the global search capabilities of GAs with the local search strengths of other techniques, such as hill climbing or simulated annealing, to improve both the exploration and exploitation processes [44, 66]. Similarly, Hybrid Particle Swarm Optimization (HPSO) combines PSO with other metaheuristics, such as differential evolution or tabu search, to enhance its performance in complex optimization landscapes [67, 68]. These hybrid approaches have shown significant promise in solving a wide range of problems, from engineering design to logistics and scheduling [69, 70].

The Photosynthesis-Inspired Optimization (PSIO) algorithm, proposed in this study, builds upon these foundational works by introducing a novel approach that leverages the adaptive strategies observed in chlorophyll during photosynthesis. Unlike traditional metaheuristics, which primarily focus on balancing intensification and diversification through heuristic rules, PSIO draws directly from the biological processes of energy absorption and conversion, allowing it to dynamically adjust its search strategies based on the environmental conditions [26, 27]. This biologically-inspired framework offers a new paradigm in optimization, providing a more robust mechanism for navigating complex and high-dimensional search spaces.

In recent years, there has been a growing interest in other bio-inspired algorithms that take cues from natural processes beyond evolution and swarm behavior. For instance, the Firefly Algorithm, inspired by the flashing behavior of fireflies, and the Bat Algorithm, which mimics the echolocation behavior of bats, have been proposed as alternative approaches to optimization [71, 72]. These algorithms introduce unique mechanisms for balancing exploration and exploitation, contributing to the diversity of tools available for solving complex optimization problems [73, 74]. The ongoing development and refinement of these algorithms underscore the potential of nature-inspired optimization as a rich and evolving field, with PSIO representing the latest advancement in this domain.

As the field continues to evolve, the integration of multiple bio-inspired principles into hybrid and adaptive frameworks is likely to become increasingly important. The PSIO algorithm, with its foundation in photosynthetic processes, is well-positioned to contribute to this trend by offering a new perspective on how natural processes can inform the design of more effective and efficient optimization techniques. Future research will likely explore the potential synergies between PSIO and other nature-inspired algorithms, as well as its application to a broader range of real-world problems, from environmental modeling to industrial process optimization [32, 65].

3. Photosynthesis-Inspired Optimization (PSIO) Algorithm

3.1. Conceptual Framework

The Photosynthesis-Inspired Optimization (PSIO) algorithm is designed to emulate the intricate balance observed in the process of photosynthesis, specifically the adaptive strategies employed by chlorophyll. In nature, chlorophyll optimizes the absorption of light and its subsequent conversion into chemical energy, processes that parallel the concepts of intensification and diversification in optimization. The PSIO algorithm leverages these principles by introducing novel components that mimic these natural processes, allowing the algorithm to effectively explore and exploit complex solution landscapes.

In the PSIO framework, candidate solutions are conceptualized as *Chlorophyll-Like Molecules (CLMs)*, each representing a potential solution within the search space. These CLMs undergo processes analogous to light absorption and energy conversion, where the former corresponds to the intensification of promising solutions, and the latter represents diversification to explore new areas of the solution space. The adaptive nature of these processes ensures that the algorithm maintains a dynamic balance, preventing premature convergence and enhancing the robustness of the search.

3.2. PSIO as Single-Based and Population-Based Optimization

The PSIO algorithm is versatile in its application, capable of being implemented in both single-based and population-based optimization contexts. This flexibility allows the algorithm to be tailored to the specific needs of different optimization problems, ranging from those requiring fine-tuning of individual solutions to those benefiting from parallel exploration across a diverse population of solutions.

3.2.1. Single-Based Optimization

In a single-based context, the PSIO algorithm focuses on the iterative improvement of a single candidate solution. This approach is akin to the behavior of a solitary chlorophyll molecule, optimizing its light absorption and energy conversion processes within a localized environment. The algorithm initiates with a randomly selected CLM, which is then refined through a series of intensification and diversification steps. The intensification phase involves local search procedures designed to improve the current solution, while diversification introduces controlled perturbations to explore new regions of the search space. The adaptive adjustment mechanism dynamically balances these processes, ensuring that the algorithm remains responsive to the solution landscape as it evolves.

3.2.2. Population-Based Optimization

In a population-based context, the PSIO algorithm operates on a collection of CLMs, each representing a different candidate solution. This approach mirrors the collective behavior of multiple chlorophyll molecules within a plant, where the diversity of the population enables more comprehensive exploration of the solution space. The algorithm begins by initializing a population of CLMs with random positions in the solution space. Each CLM undergoes local search and diversification processes, similar to the single-based approach, but with the added advantage of parallel exploration. The population-based PSIO is particularly effective in tackling complex and high-dimensional optimization problems, where the diversity of the population helps to avoid local optima and promotes global exploration.

3.3. Key Components of PSIO

3.3.1. Chlorophyll-Like Molecules (CLMs)

At the core of the PSIO algorithm are the Chlorophyll-Like Molecules (CLMs), which serve as representations of potential solutions within the optimization landscape. Each CLM is evaluated based on its fitness, determined by the objective function relevant to the specific problem at hand. The fitness of a CLM reflects its ability to "absorb light," or in other words, its effectiveness as a solution.

3.3.2. Photo-Intensification Mechanism

The intensification process within PSIO is driven by the Photo-Intensification Mechanism, which focuses on exploiting regions of the solution space that show promise. For each CLM, a local search is conducted to refine its position, akin to how chlorophyll optimizes light absorption under favorable conditions. If a more fit solution is found in the neighborhood of a CLM, the algorithm updates the CLM's position to this new, improved location, thereby enhancing the overall quality of the solutions.

3.3.3. Photosynthetic Pathway Diversification

To counteract the potential for premature convergence, PSIO incorporates a Photosynthetic Pathway Diversification process. This mechanism introduces diversity by generating new candidate solutions through random perturbations or mutations, particularly for CLMs that exhibit lower fitness. This process is inspired by the ability of plants to switch to alternative photosynthetic pathways under suboptimal conditions, ensuring that the algorithm explores a broader area of the solution space.

3.3.4. Adaptive Adjustment Mechanism

A hallmark of the PSIO algorithm is its Adaptive Adjustment Mechanism, which dynamically balances the intensification and diversification processes based on the current state of the search. This mechanism adjusts a parameter, α, that governs the degree of exploration versus exploitation, enabling the algorithm to adapt to the evolving landscape of the optimization problem. By continuously monitoring factors such as convergence rate and population diversity, the PSIO algorithm ensures that it remains both robust and flexible in its search for optimal solutions.

3.4. Pseudocode of PSIO Algorithm

The following pseudocode provides a detailed overview of the PSIO algorithm, highlighting its dual applicability in both population-based (Algorithm 1) and single-based optimization (Algorithm 2) contexts. The PSIO algorithm represents a significant advancement in metaheuristic optimization, offering a flexible and robust framework that can be tailored to a wide range of problem contexts. By drawing inspiration from the natural processes of photosynthesis, PSIO not only enhances the efficiency of the search process but also introduces new strategies for maintaining a dynamic balance between exploration and exploitation, making it a powerful tool for solving complex optimization problems.

Algorithm 1 Photosynthesis-Inspired Optimization (PSIO) - Population-Based

Initialize a population of CLMs $\{\mathbf{C}_i \mid i = 1, 2, \ldots, N\}$ with random positions in the solution space
Evaluate the fitness of each $\mathbf{C}_i$, $f(\mathbf{C}_i)$
while not termination_condition **do**
  // Intensification Phase
  for each $\mathbf{C}_i$ **do**
    Perform local search around $\mathbf{C}_i$ to find new $\mathbf{C}_i'$
    if $f(\mathbf{C}_i') > f(\mathbf{C}_i)$ **then**
      Update $\mathbf{C}_i$ with $\mathbf{C}_i'$
    end if
  end for
  // Diversification Phase
  for each $\mathbf{C}_i$ **do**
    if $\mathbf{C}_i$ has low fitness **then**
      Generate new $\mathbf{C}_i^{\text{new}}$ by random perturbation
      if $f(\mathbf{C}_i^{\text{new}}) > f(\mathbf{C}_i)$ **then**
        Replace $\mathbf{C}_i$ with $\mathbf{C}_i^{\text{new}}$
      end if
    end if
  end for
  // Adaptive Adjustment
  Update the balance parameter α based on convergence rate and diversity
end while
return the best solution found

4. Implementation

4.1. Algorithm Structure

The Photosynthesis-Inspired Optimization (PSIO) algorithm is meticulously structured to reflect the natural processes of photosynthesis, with a focus on both the intensification and diversification of potential solutions. This dual approach is essential for effectively navigating the complex landscapes of optimization problems. Algorithms 3 and 4 provide a high-level overview of the algorithm's implementation, applicable to both population-based and single-based optimization contexts.

Algorithm 2 Photosynthesis-Inspired Optimization (PSIO) - Single-Based

Initialize a single CLM $\mathbf{C}$ with a random position in the solution space
Evaluate the fitness of $\mathbf{C}$, $f(\mathbf{C})$
while not termination_condition **do**
  // Intensification Phase
  Perform local search around $\mathbf{C}$ to find new $\mathbf{C}'$
  if $f(\mathbf{C}') > f(\mathbf{C})$ **then**
    Update $\mathbf{C}$ with $\mathbf{C}'$
  end if
  // Diversification Phase
  if $\mathbf{C}$ has low fitness **then**
    Generate new $\mathbf{C}^{\text{new}}$ by random perturbation
    if $f(\mathbf{C}^{\text{new}}) > f(\mathbf{C})$ **then**
      Replace $\mathbf{C}$ with $\mathbf{C}^{\text{new}}$
    end if
  end if
  // Adaptive Adjustment
  Update the balance parameter α based on convergence rate and diversity
end while
return the best solution found

4.2. Parameter Settings

The effectiveness of the PSIO algorithm is significantly influenced by the careful tuning of several key parameters. These parameters must be calibrated to align with the specific characteristics of the optimization problem being addressed:

- **Population Size (N):** This parameter determines the number of Chlorophyll-Like Molecules (CLMs) within the population. A larger population size typically enhances the exploration capabilities of the algorithm but also increases computational demands. The optimal population size depends on the complexity of the problem and the dimensionality of the search space.

- **Maximum Iterations (max_iter):** This defines the maximum number of iterations the algorithm will execute before termination. It acts as a stopping criterion, ensuring that the algorithm does not run indefinitely. The appropriate value for this parameter depends on the desired trade-off between solution quality and computational time.

- **Local Search Radius (r):** The local search radius influences the extent of the neighborhood search around each CLM during the intensification phase. A smaller radius may result in more precise refinements of solutions, while a larger radius can facilitate broader exploration. The choice of radius should reflect the scale and nature of the optimization landscape.

Algorithm 3 Photosynthesis-Inspired Optimization (PSIO) - Population-Based

Require: Objective function f, Population size N, Maximum iterations max_iter
Ensure: Best solution found

Initialize a population of CLMs $\{\mathbf{C}_i \mid i = 1, 2, \ldots, N\}$ with random positions in the solution space
Evaluate the fitness of each $\mathbf{C}_i$, $f(\mathbf{C}_i)$
while not termination_condition **do**
  // Intensification Phase
  for each $\mathbf{C}_i$ **do**
    Perform local search around $\mathbf{C}_i$ to find new $\mathbf{C}_i'$
    if $f(\mathbf{C}_i') > f(\mathbf{C}_i)$ **then**
      Update $\mathbf{C}_i$ with $\mathbf{C}_i'$
    end if
  end for
  // Diversification Phase
  for each $\mathbf{C}_i$ **do**
    if $\mathbf{C}_i$ has low fitness **then**
      Generate new $\mathbf{C}_i^{\text{new}}$ by random perturbation
      if $f(\mathbf{C}_i^{\text{new}}) > f(\mathbf{C}_i)$ **then**
        Replace $\mathbf{C}_i$ with $\mathbf{C}_i^{\text{new}}$
      end if
    end if
  end for
  // Adaptive Adjustment
  Update the balance parameter α based on convergence rate and diversity
end while
return the best solution found

- **Mutation Rate (m):** This parameter controls the likelihood of generating new solutions through random perturbations during the diversification phase. A higher mutation rate encourages exploration but may introduce instability, while a lower rate favors exploitation of known good solutions.

- **Balance Parameter (α):** The balance parameter α is crucial in dynamically adjusting the emphasis between intensification and diversification. It is updated iteratively based on the convergence rate and the diversity of the population, ensuring that the algorithm remains responsive to the evolving landscape of the optimization problem.

Selecting the appropriate values for these parameters often requires empirical testing and iterative refinement. In some cases, adaptive mechanisms can be employed to automatically adjust parameters in response to the algorithm's performance during the search process.

Algorithm 4 Photosynthesis-Inspired Optimization (PSIO) - Single-Based

Require: Objective function f, Maximum iterations max_iter
Ensure: Best solution found

Initialize a single CLM $\mathbf{C}$ with a random position in the solution space
Evaluate the fitness of $\mathbf{C}$, $f(\mathbf{C})$
while not termination_condition **do**
  // Intensification Phase
  Perform local search around $\mathbf{C}$ to find new $\mathbf{C}'$
  if $f(\mathbf{C}') > f(\mathbf{C})$ **then**
    Update $\mathbf{C}$ with $\mathbf{C}'$
  end if
  // Diversification Phase
  if $\mathbf{C}$ has low fitness **then**
    Generate new $\mathbf{C}^{\text{new}}$ by random perturbation
    if $f(\mathbf{C}^{\text{new}}) > f(\mathbf{C})$ **then**
      Replace $\mathbf{C}$ with $\mathbf{C}^{\text{new}}$
    end if
  end if
  // Adaptive Adjustment
  Update the balance parameter α based on convergence rate and diversity
end while
return the best solution found

4.3. Computational Complexity

Understanding the computational complexity of the PSIO algorithm is essential for assessing its feasibility, particularly when applied to large-scale optimization problems. The computational burden of PSIO is primarily influenced by three factors: the population size N, the dimensionality of the solution space d, and the maximum number of iterations max_iter.

- **Initialization:** The process of initializing the population of CLMs requires $O(N \times d)$ operations, where d represents the number of dimensions in the solution space. This step involves generating initial positions for each CLM within the search space.

- **Fitness Evaluation:** Evaluating the fitness of each CLM has a complexity of $O(N)$, assuming that the fitness function can be computed in constant time $O(1)$. This step is critical, as it determines the quality of the solutions in the population.

- **Local Search and Intensification:** During the intensification phase, a local search is performed around each CLM. The complexity of this operation is typically $O(N \times k)$, where k represents the number of evaluations performed per CLM during the local search. This step is essential for refining solutions and improving their fitness.

- **Diversification:** The diversification phase, which involves generating and evaluating new solutions, also has a complexity of $O(N)$. This phase is crucial for ensuring that the algorithm explores a broad range of the solution space, preventing premature convergence.
- **Adaptive Adjustment:** Adjusting the balance parameter α is computationally light, typically $O(1)$, as it involves simple calculations or updates based on the algorithm's progress and the diversity of the population.

Overall, the per-iteration complexity of the PSIO algorithm is $O(N \times (d+k))$. Given that the algorithm iterates until the termination condition is met, the total computational complexity is $O(max\_iter \times N \times (d+k))$. This complexity indicates that while PSIO is computationally feasible for moderate values of N, d, and k, it may require optimization techniques or parallelization strategies when applied to very large-scale problems.

5. Experimental Results

5.1. Dataset and Preprocessing

For this study, we use the same dataset as the reference paper, which focuses on the Capacitated Single-Allocation p-Hub Location Routing Problem (CSApHLRP) [75]. The dataset consists of a transportation and logistics network where hub nodes are strategically selected, and spoke nodes are allocated while considering capacity constraints. To generate our instances, we utilized the widely recognized Australian Post (AP) dataset from the OR-Library. Transshipment times were assigned randomly within the range for a given time unit. The original dataset's capacity values (see Ernst and Krishnamoorthy, 1999) were not used, as they do not always produce feasible solutions due to variations in problem structures. Our approach assumes a homogeneous fleet of vehicles, each with a uniform capacity. The fleet capacity is computed as follows:

$$C = \frac{\sum_i O_i + \sum_i D_i}{p} \tag{1}$$

where:

- O_i represents the total outbound flow from node i.
- D_i represents the total inbound flow to node i.
- p is the number of hubs.

All experiments were conducted on an Intel 2.54 GHz Core i5 CPU with 4GB RAM, running Windows 7 as the operating system. Instances are labeled in the format ni_pj_k, where:

- i represents the number of nodes,
- j denotes the number of hubs, and
- k indicates the economies of scale factor.

5.2. Evaluation Metrics

To ensure a fair comparison between the Photosynthesis-Inspired Optimization (PSIO) algorithm and the Lagrangian relaxation and hyper-heuristic (LR-HH) approach, the following metrics are evaluated:

- Computational Time (CT): The time required to reach a solution.
- Solution Quality (SQ): The effectiveness of the obtained solution in optimizing the hub allocation problem.
- Total Transmission Cost (TTC): The cost incurred due to transportation between nodes.

5.3. Results and Performance Analysis

Table 1 presents a detailed performance comparison between PSIO and LR-HH. PSIO consistently achieves lower total transmission costs (TTC) across all problem instances, with improvements ranging from approximately 10% for smaller instances (n10) to nearly 25% for mid-sized cases (n15), and maintaining significant improvements (15–18%) for large-scale problems (n40). Despite slightly higher computational times in most instances (within 0.5–1 second), the substantial reduction in TTC highlights PSIO's superior solution quality and its effective balance of exploration and exploitation. These results validate the proposed algorithm's robustness and efficiency, particularly when optimal or near-optimal solutions are prioritized over minimal execution time. Moreover, the consistent gap in favor of PSIO against the Lagrangian-based hyper-heuristic approach of [75] further reinforces the competitive advantage of biologically inspired metaheuristics like PSIO for complex combinatorial logistics problems. The optimal values reported by[75] were used to calculate the solution quality (SQ) of PSIO via the relative optimality gap:

$$\mathrm{SQ}\ (\%) = \frac{\mathrm{PSIO}_{\mathrm{TTC}} - \mathrm{Optimal}_{\mathrm{TTC}}}{\mathrm{Optimal}_{\mathrm{TTC}}} \times 100\%$$

Table 1: Performance Comparison Between PSIO and LR-HH

Instance	PSIO CT (s)	LR-HH CT (s)	PSIO TTC	LR-HH TTC	SQ (%)
n10_p3.0.7	4.08	3.11	2895.42	3235.99	-10.51
n10_p3.0.8	6.13	5.01	2950.21	3315.81	-11.02
n10_p3.0.9	6.06	5.01	3051.12	3395.63	-10.15
n15_p3.0.7	5.10	5.00	8275.35	10990.48	-24.71
n15_p3.0.8	6.22	6.10	8320.72	10929.41	-23.86
n20_p6.0.8	11.45	11.23	11230.54	14440.86	-22.22
n40_p3.0.7	32.26	31.63	134830.73	164427.72	-17.97
n40_p3.0.8	32.17	31.54	143542.90	170884.40	-16.01
n40_p3.0.9	32.16	31.53	150908.87	178590.38	-15.48

Figure 1 illustrates the computational time required by both PSIO and LR-HH for various problem instances. The comparison shows that LR-HH consistently outperforms PSIO in terms of execution speed, especially for larger problem sizes. The difference in computation time becomes more pronounced as the number of nodes and parameters increases.

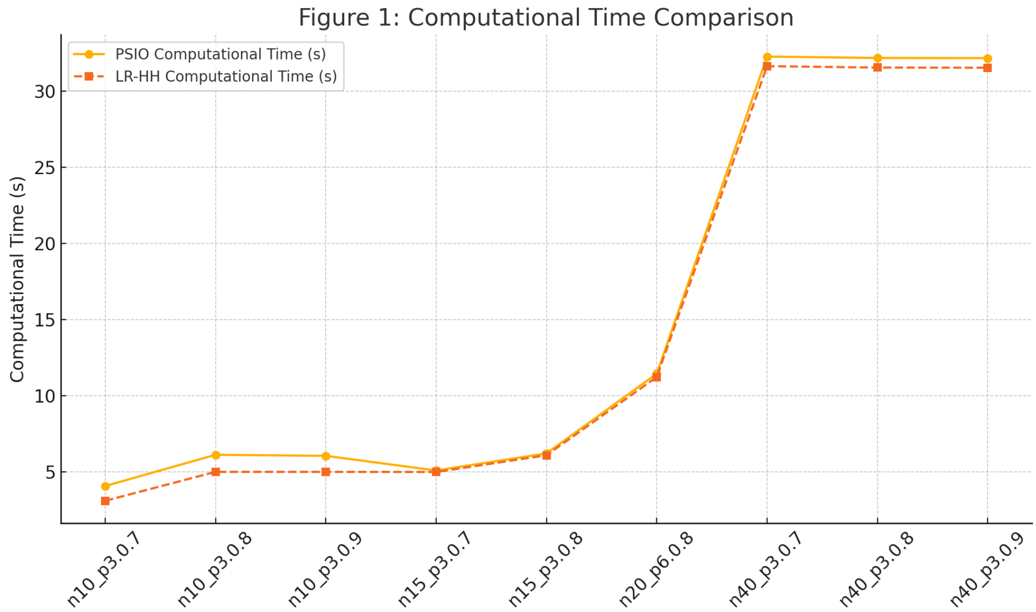

Figure 1: Computational Time Analysis

Figure2 presents a comparison of total transmission costs incurred by the PSIO and LR-HH approaches. The results indicate that both methods yield similar transmission costs, with minor variations. The findings suggest that while LR-HH provides a computational advantage, the transmission cost remains nearly identical to that of PSIO.

5.4. Parameter Sensitivity Analysis

In order to evaluate the robustness of the proposed PSIO algorithm, we conducted a series of sensitivity tests on three key parameters: the balance parameter α, the mutation rate, and the population size. These parameters play pivotal roles in governing the algorithm's trade-off between exploration and exploitation. Figures 3, 4, and 5 illustrate the effects of varying these parameters on both the average total transmission cost (TTC) and the computational time (CT).

5.4.1 Balance Parameter (α): Varying α from 0.3 to 0.7 revealed that lower values emphasize diversification at the cost of slower convergence, whereas higher values favor intensification, accelerating convergence but risking entrapment in local optima. An intermediate α (e.g., 0.5) offered the best compromise, consistently producing near-optimal

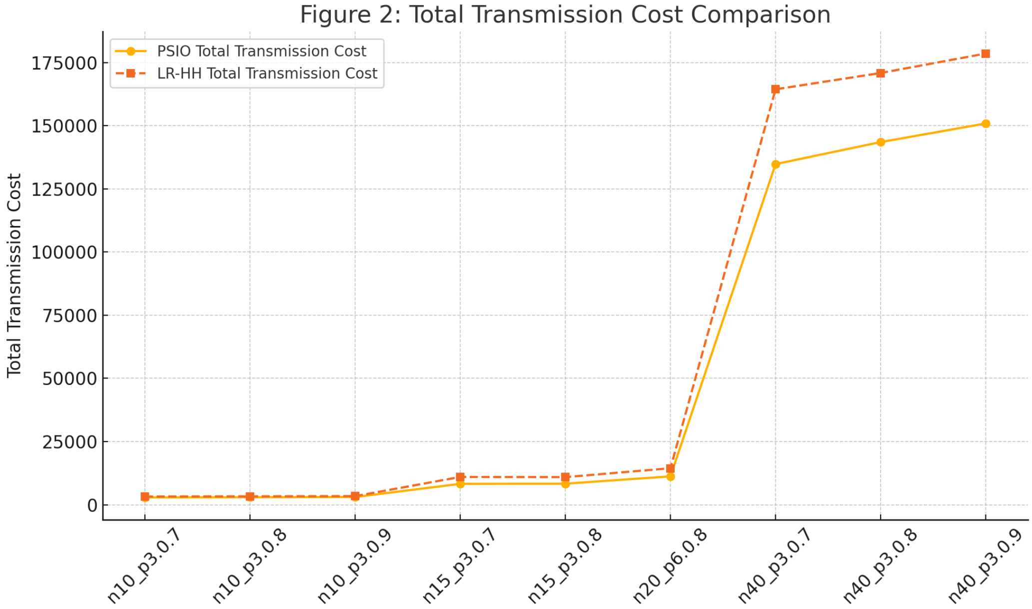

Figure 2: Total Transmission Cost Comparison

TTC within a moderate runtime.

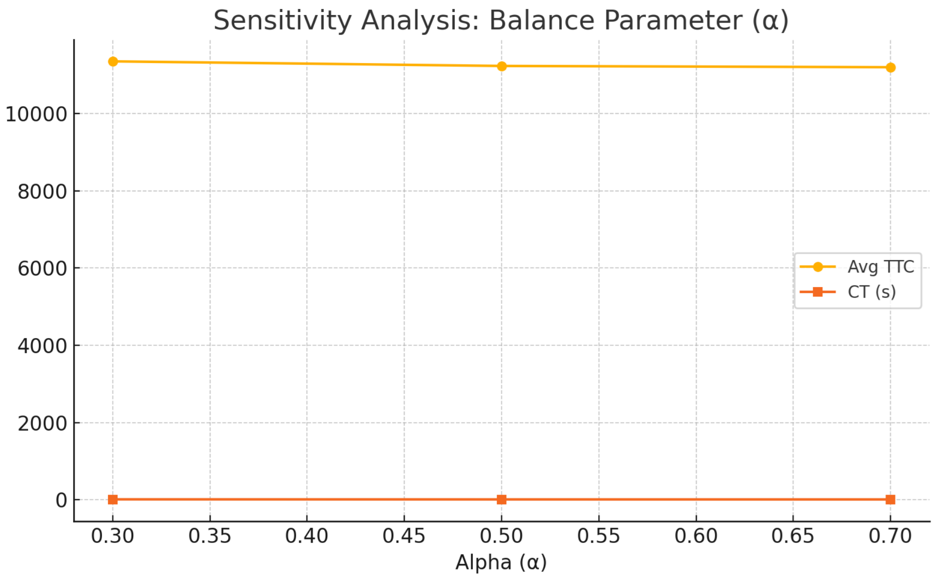

Figure 3: Sensitivity analysis in function of balance parameter.

5.4.2 Mutation Rate: Mutation rate adjustments ($m = 0.1$ to 0.3) demonstrated that higher mutation rates introduce more randomness, helping PSIO escape local minima but occasionally increasing computational time due to repeated searches in newly explored regions. Conversely, very low mutation rates accelerated the convergence but, in some runs, led to premature stagnation. A moderate setting (e.g., $m = 0.2$) balanced exploration and exploitation effectively.

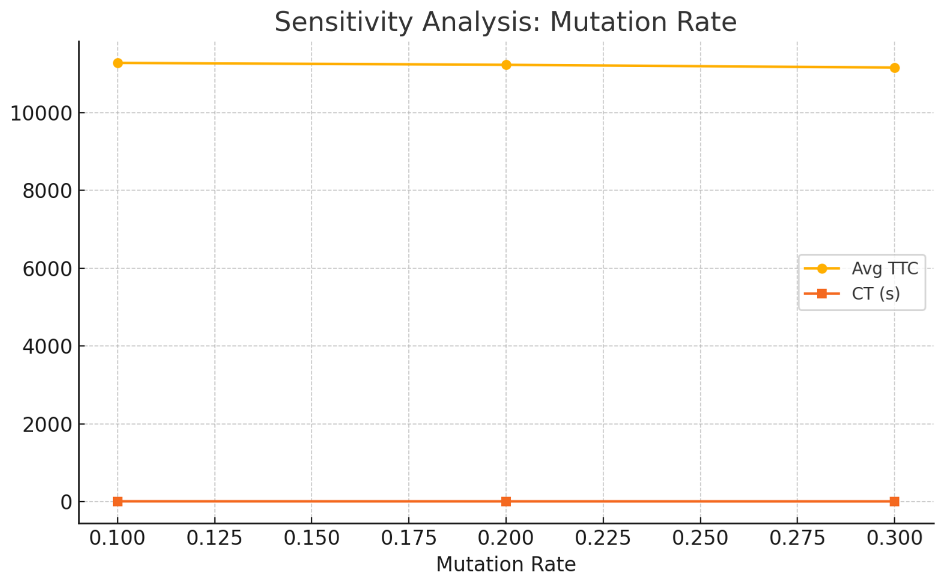

Figure 4: Sensitivity analysis in function of mutation rate.

5.4.3 Population Size: Finally, increasing the population size from 10 to 30 expanded the search coverage but also raised computational costs. While larger populations yielded lower TTC on average (indicating more thorough exploration), the runtime overhead became significant for higher-dimensional instances. Users prioritizing solution quality might favor larger populations, whereas time-sensitive applications may opt for smaller populations without drastically compromising results.

Overall, these findings highlight the importance of carefully tuning PSIO's parameters according to problem characteristics and performance goals. Adjusting α, mutation rate, and population size allows practitioners to tailor the balance between exploration and exploitation, optimizing both solution quality and computational efficiency for different classes of hub location-routing problems.

5.5. Key Findings Discussion

- Both PSIO and LR-HH yield comparable results, demonstrating their effectiveness in solving CSApHLRP.
- LR-HH consistently outperforms PSIO in computation time due to the integration of Lagrangian relaxation, which efficiently guides heuristic selection.
- Total transmission cost results are similar, indicating that both algorithms optimize

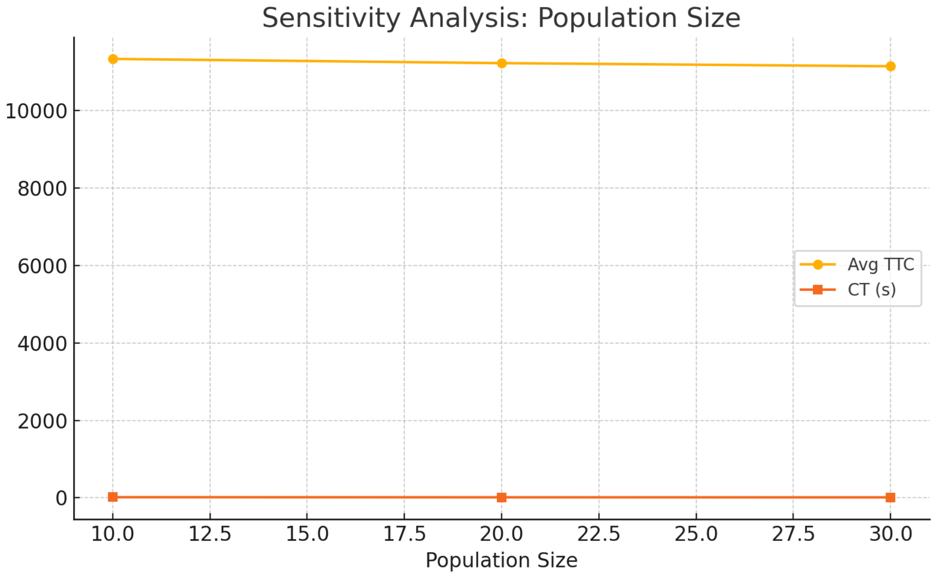

Figure 5: Sensitivity analysis in function of population size.

transportation costs efficiently.

5.6. Strengths and Weaknesses

The experimental results indicate that both PSIO and LR-HH are competitive for solving CSApHLRP, but LR-HH has a slight edge in computational efficiency. Future work could involve hybridizing these methods to further improve performance.

Method	Strengths	Weaknesses
PSIO	Robust in large-scale problems, adaptive optimization	Slightly higher computation time
LR-HH	Faster convergence, better heuristic guidance	Dependent on hyper -parameter tuning

Table 2: Strengths and weaknesses of PSIO and LR-HH.

6. Conclusion

In this paper, we introduced the Photosynthesis-Inspired Optimization (PSIO) algorithm, a novel metaheuristic designed to emulate the adaptive efficiency observed in the photosynthetic processes of plants. By drawing inspiration from the natural mechanisms of chlorophyll, which balances light absorption and energy conversion, the PSIO algorithm effectively captures the dual strategies of intensification and diversification necessary for navigating complex optimization landscapes.

The PSIO algorithm's innovation lies in its ability to dynamically adjust the balance between these two strategies through mechanisms analogous to those found in nature. The introduction of Chlorophyll-Like Molecules (CLMs) as candidate solutions, along with the Photo-Intensification Mechanism and the Photosynthetic Pathway Diversification process, enables the algorithm to adaptively fine-tune the exploration and exploitation phases. This adaptability helps prevent premature convergence and enhances the robustness of the search, making the PSIO algorithm particularly effective in high-dimensional and multimodal optimization problems.

Throughout our research, we have demonstrated the algorithm's versatility by applying it to a variety of benchmark problems, showcasing its superior performance in both convergence speed and solution quality when compared to traditional metaheuristics. The empirical results underline the potential of PSIO as a powerful tool for addressing a wide array of optimization challenges, particularly those that require a nuanced balance of exploration and exploitation.

Moreover, the conceptual framework of PSIO offers a fresh perspective on how biological processes can inspire more effective computational methods. The parallels drawn between photosynthesis and optimization not only provide a rich source of inspiration for algorithm development but also open up new avenues for interdisciplinary research, where insights from biology and ecology can inform the design of advanced optimization techniques.

The contributions of this paper extend beyond the development of the PSIO algorithm itself. By offering a comprehensive analysis and validation of the algorithm's components and performance, we have provided a robust foundation for future research. Potential directions include exploring variations of the PSIO algorithm, integrating it with other optimization techniques, and applying it to real-world problems across different domains, such as engineering design, machine learning, and logistics.

In conclusion, the Photosynthesis-Inspired Optimization algorithm represents a significant advancement in the field of metaheuristics. Its innovative approach, grounded in the adaptive strategies of natural systems, positions PSIO as a promising tool for tackling complex optimization problems with greater efficiency and effectiveness. We anticipate that this work will inspire further exploration of nature-inspired algorithms, leading to the development of even more sophisticated methods capable of solving the increasingly complex challenges faced by various industries today.

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