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Upper and Lower Contra- (τ_1, τ_2) -continuity

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Abstract. This paper presents new classes of multifunctions between bitopological spaces, namely upper contra- (τ_1, τ_2) -continuous multifunctions and lower contra- (τ_1, τ_2) -continuous multifunctions. Moreover, several characterizations and some properties concerning upper contra- (τ_1, τ_2) -continuous multifunctions and lower contra- (τ_1, τ_2) -continuous multifunctions are investigated.

2020 Mathematics Subject Classifications: 54C08, 54C60

Key Words and Phrases: Upper contra- (τ_1, τ_2) -continuous multifunction, lower contra- (τ_1, τ_2) continuous multifunction

1. Introduction

Weaker and stronger forms of open sets in topological spaces such as semi-open sets, preopen sets, α -open sets, β -open sets, δ -open sets and θ -open sets play an important role in the researches of generalizations of continuity. By using these sets many authors introduced and investigated various types of continuity. Viriyapong and Boonpok [1] investigated some characterizations of (Λ, sp) -continuous functions by utilizing the notions of (Λ, sp) -open sets and (Λ, sp) -closed sets due to Boonpok and Khampakdee [2]. Dungthaisong et al. [3] introduced and studied the concept of $g_{(m,n)}$ -continuous functions. Duangphui et al. [4] introduced and investigated the notion of $(\mu, \mu')^{(m,n)}$ continuous functions. Furthermore, several characterizations of almost (Λ, p) -continuous functions, strongly $\theta(\Lambda, p)$ -continuous functions, almost strongly $\theta(\Lambda, p)$ -continuous functions, $(\Lambda, p(\star))$ -continuous functions, weakly (Λ, b) -continuous functions, $\theta(\star, p)$ -continuous functions, $(\Lambda, p(\star))$ -continuous functions, pairwise almost M-continuous functions, (τ_1, τ_2) continuous functions, almost (τ_1, τ_2) -continuous functions, weakly (τ_1, τ_2) -continuous functions and slightly $(\tau_1, \tau_2)s$ -continuous functions were presented in [5], [6], [7], [8], [9],

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[10], [11], [12], [13], [14], [15], [16], [17], [18] and [19], respectively. Kong-ied at al. [20] introduced and studied the concept of almost quasi (τ_1, τ_2) -continuous functions. Chiangpradit et al. [21] introduced and investigated the notion of weakly quasi (τ_1, τ_2) continuous functions. Thongmoon et al. [22] introduced and studied the notion of rarely (τ_1, τ_2) -continuous functions. Srisarakham et al. [23] introduced and investigated the concept of faintly (τ_1, τ_2) -continuous functions. On the other hand, the present authors introduced and studied the notions of $\delta(\tau_1, \tau_2)$ -continuous functions [24], quasi $\theta(\tau_1, \tau_2)$ -continuous functions [25], almost weakly (τ_1, τ_2) -continuous functions [26] and almost nearly (τ_1, τ_2) -continuous functions [27]. In 1966, Dontchev [28] introduced the notion of contra-continuity in topological spaces. Dontchev and Noiri [29] introduced and studied the concept of *RC*-continuity between topological spaces which is weaker than contra-continuity. Jafari and Noiri [30] introduced a new class of function called contraprecontinuous functions which is weaker than contra-continuous functions and studied several basic properties of contra-precontinuous functions. Ekici [31] introduced and studied a new class of functions called almost contra-precontinuous functions which generalize classes of regular set-connected functions [32], contra-precontinuous functions [30], contra-continuous functions [28], almost s-continuous functions [33] and perfectly continuous functions [34].

In 2008, Ekici et al. [35] extended the notion of contra-continuous functions to the setting of multifunctions. Noiri and Popa [36] introduced the notion of weakly precontinuous multifunctions. Moreover, several characterizations and some properties concerning $(\tau_1, \tau_2)\delta$ -semicontinuous multifunctions, almost weakly (τ_1, τ_2) -continuous multifunctions, weakly quasi (Λ , sp)-continuous multifunctions, \star -continuous multifunctions, $\beta(\star)$ continuous multifunctions, α -*-continuous multifunctions, almost α -*-continuous multifunctions, almost quasi \star -continuous multifunctions, weakly α - \star -continuous multifunctions, $s\beta(\star)$ -continuous multifunctions, weakly $s\beta(\star)$ -continuous multifunctions, $\theta(\star)$ -quasi continuous multifunctions, almost i^{*}-continuous multifunctions, weakly (Λ, sp) -continuous multifunctions, $\alpha(\Lambda, sp)$ -continuous multifunctions, almost $\alpha(\Lambda, sp)$ -continuous multifunctions, weakly $\alpha(\Lambda, sp)$ -continuous multifunctions, almost $\beta(\Lambda, sp)$ -continuous multifunctions, slightly (Λ, sp) -continuous multifunctions, (τ_1, τ_2) -continuous multifunctions, almost (τ_1, τ_2) -continuous multifunctions, weakly (τ_1, τ_2) -continuous multifunctions, weakly quasi (τ_1, τ_2) -continuous multifunctions, almost quasi (τ_1, τ_2) -continuous multifunctions, c- (τ_1, τ_2) -continuous multifunctions, c-quasi (τ_1, τ_2) -continuous multifunctions, s- $(\tau_1, \tau_2)p$ continuous multifunctions, slightly $\alpha(\tau_1, \tau_2)$ -continuous multifunctions and slightly $(\tau_1, \tau_2)p$ continuous multifunctions were established in [37], [38], [39], [40], [41], [42], [43], [44], [45], [46], [47], [48], [49], [50], [51], [52], [53], [54], [55], [56], [57], [58], [59], [60], [61], [62], [63], [63], [64], [65], [66][64] and [65], respectively. On the other hand, the present authors introduced and investigated the notions of rarely s- (τ_1, τ_2) p-continuous multifunctions [66], almost nearly (τ_1, τ_2) continuous multifunctions [67], s- (τ_1, τ_2) -continuous multifunctions [68], quasi $\theta(\tau_1, \tau_2)$ continuous multifunctions [69], almost nearly quasi (τ_1, τ_2) -continuous multifunctions [70], weakly s- (τ_1, τ_2) -continuous multifunctions [71], nearly (τ_1, τ_2) -continuous multifunctions [72] and almost quasi (τ_1, τ_2) -continuous multifunctions [73]. Ekici et al. [74] introduced and studied two new concepts namely contra-precontinuous multifunctions and almost

contra-precontinuous multifunctions which are containing the class of contra-continuous multifunctions [35] and contained in the class of weakly precontinuous multifunctions. Ekici et al. [75] introduced and studied a new generalization of contra-continuous multifunctions called almost contra-continuous multifunctions. Recently, the present authors [76] introduced and investigated the notions of upper almost contra- (Λ, sp) -continuous multifunctions and lower almost contra- (Λ, sp) -continuous multifunctions. In this paper, we introduce the concepts of upper contra- (τ_1, τ_2) -continuous multifunctions. We also investigate several characterizations of upper contra- (τ_1, τ_2) -continuous multifunctions and lower contra- (τ_1, τ_2) -continuous multifunctions.

2. Preliminaries

Throughout the present paper, spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2) (or simply X and Y) always mean bitopological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a bitopological space (X, τ_1, τ_2) . The closure of A and the interior of A with respect to τ_i are denoted by τ_i -Cl(A) and τ_i -Int(A), respectively, for i = 1, 2. A subset A of a bitopological space (X, τ_1, τ_2) is called $\tau_1 \tau_2$ -closed [77] if $A = \tau_1$ -Cl(τ_2 -Cl(A)). The complement of a $\tau_1 \tau_2$ -closed set is called $\tau_1 \tau_2$ -open. The intersection of all $\tau_1 \tau_2$ -closed sets of X containing A is called the $\tau_1 \tau_2$ -closure [77] of A and is denoted by $\tau_1 \tau_2$ -Cl(A). The union of all $\tau_1 \tau_2$ -open sets of X contained in A is called the $\tau_1 \tau_2$ -closure [77] of A and is denoted by $\tau_1 \tau_2$ -Cl(A).

Lemma 1. [77] Let A and B be subsets of a bitopological space (X, τ_1, τ_2) . For the $\tau_1 \tau_2$ closure, the following properties hold:

- (1) $A \subseteq \tau_1 \tau_2 Cl(A)$ and $\tau_1 \tau_2 Cl(\tau_1 \tau_2 Cl(A)) = \tau_1 \tau_2 Cl(A)$.
- (2) If $A \subseteq B$, then $\tau_1 \tau_2$ -Cl(A) $\subseteq \tau_1 \tau_2$ -Cl(B).
- (3) $\tau_1\tau_2$ -Cl(A) is $\tau_1\tau_2$ -closed.
- (4) A is $\tau_1 \tau_2$ -closed if and only if $A = \tau_1 \tau_2$ -Cl(A).
- (5) $\tau_1 \tau_2 Cl(X A) = X \tau_1 \tau_2 Int(A).$

A subset A of a bitopological space (X, τ_1, τ_2) is called $\alpha(\tau_1, \tau_2)$ -open [78] if $A \subseteq \tau_1\tau_2$ -Int $(\tau_1\tau_2$ -Cl $(\tau_1\tau_2$ -Int(A))). The complement of an $\alpha(\tau_1, \tau_2)$ -open set is called $\alpha(\tau_1, \tau_2)$ closed. A subset A of a bitopological space (X, τ_1, τ_2) is called $(\tau_1, \tau_2)r$ -open [79] (resp. $(\tau_1, \tau_2)s$ -open [37], $(\tau_1, \tau_2)p$ -open [37], $(\tau_1, \tau_2)\beta$ -open [37]) if $A = \tau_1\tau_2$ -Int $(\tau_1\tau_2$ -Cl(A))(resp. $A \subseteq \tau_1\tau_2$ -Cl $(\tau_1\tau_2$ -Int $(A)), A \subseteq \tau_1\tau_2$ -Int $(\tau_1\tau_2$ -Cl $(A)), A \subseteq \tau_1\tau_2$ -Cl $(\tau_1\tau_2$ -Int $(\tau_1\tau_2$ -Cl(A))))). The complement of a $(\tau_1, \tau_2)r$ -open (resp. $(\tau_1, \tau_2)s$ -open, $(\tau_1, \tau_2)p$ -open, $(\tau_1, \tau_2)\beta$ -open, $\alpha(\tau_1, \tau_2)$ -open) set is called $(\tau_1, \tau_2)r$ -closed (resp. $(\tau_1, \tau_2)s$ -closed, $(\tau_1, \tau_2)p$ -closed, $(\tau_1, \tau_2)\beta$ closed, $\alpha(\tau_1, \tau_2)$ -closed). Let A be a subset of a bitopological space (X, τ_1, τ_2) . The set $\cap \{G \mid A \subseteq G \text{ and } G \text{ is } \tau_1\tau_2$ -open} \text{ is called the } \tau_1\tau_2-kernel [77] of A and is denoted by $\tau_1\tau_2$ -ker(A). **Lemma 2.** [77] For subsets A, B of a bitopological space (X, τ_1, τ_2) , the following properties hold:

- (1) $A \subseteq \tau_1 \tau_2$ -ker(A).
- (2) If $A \subseteq B$, then $\tau_1 \tau_2$ -ker $(A) \subseteq \tau_1 \tau_2$ -ker(B).
- (3) If A is $\tau_1 \tau_2$ -open, then $\tau_1 \tau_2$ -ker(A) = A.
- (4) $x \in \tau_1 \tau_2$ -ker(A) if and only if $A \cap H \neq \emptyset$ for every $\tau_1 \tau_2$ -closed set H containing x.

By a multifunction $F: X \to Y$, we mean a point-to-set correspondence from X into Y, and we always assume that $F(x) \neq \emptyset$ for all $x \in X$. For a multifunction $F: X \to Y$, we shall denote the upper and lower inverse of a set B of Y by $F^+(B)$ and $F^-(B)$, respectively, that is, $F^+(B) = \{x \in X \mid F(x) \subseteq B\}$ and $F^-(B) = \{x \in X \mid F(x) \cap B \neq \emptyset\}$. In particular, $F^{-}(y) = \{x \in X \mid y \in F(x)\}$ for each point $y \in Y$. For each $A \subseteq X$, $F(A) = \bigcup_{x \in A} F(x).$

3. Upper and lower contra- (τ_1, τ_2) -continuous multifunctions

In this section, we introduce the concepts of upper contra- (τ_1, τ_2) -continuous multifunctions and lower contra- (τ_1, τ_2) -continuous multifunctions. Furthermore, several characterizations of upper contra- (τ_1, τ_2) -continuous multifunctions and lower contra- (τ_1, τ_2) continuous multifunctions are discussed.

Definition 1. A multifunction $F: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called upper contra- (τ_1, τ_2) continuous at a point $x \in X$ if for each $\sigma_1 \sigma_2$ -closed set K of Y such that $x \in F^+(K)$, there exists a $\tau_1\tau_2$ -open set U of X containing x such that $U \subseteq F^+(K)$. A multifunction $F: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called upper contra- (τ_1, τ_2) -continuous if F is upper contra- (τ_1, τ_2) -continuous at each point x of X.

Theorem 1. For a multifunction $F: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:

- (1) F is upper contra- (τ_1, τ_2) -continuous;
- (2) $F^+(K)$ is $\tau_1\tau_2$ -open in X for every $\sigma_1\sigma_2$ -closed set K of Y;
- (3) $F^{-}(V)$ is $\tau_{1}\tau_{2}$ -closed in X for every $\sigma_{1}\sigma_{2}$ -open set V of Y;
- (4) for each $x \in X$ and each $\sigma_1 \sigma_2$ -closed set K of Y containing F(x), there exists a $\tau_1 \tau_2$ -open set U of X containing x such that if $y \in U$, then $F(y) \subseteq K$.

Proof. (1) \Leftrightarrow (2): Let K be any $\sigma_1 \sigma_2$ -closed set of Y and $x \in F^+(K)$. Since F is upper contra- (τ_1, τ_2) -continuous, there exists a $\tau_1 \tau_2$ -open set U of X containing x such that $U \subseteq F^+(K)$. Thus, $F^+(K)$ is $\tau_1 \tau_2$ -open in X. The converse of the proof is similar. (2) \Leftrightarrow (3): This follows from the fact that $F^+(Y-B) = X - F^-(B)$ for every subset

 $B ext{ of } Y.$

 $(1) \Leftrightarrow (4)$: Obvious.

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Definition 2. A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called lower contra- (τ_1, τ_2) continuous at a point $x \in X$ if for each $\sigma_1 \sigma_2$ -closed set K of Y such that $x \in F^-(K)$, there exists a $\tau_1 \tau_2$ -open set U of X containing x such that $U \subseteq F^-(K)$. A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called lower contra- (τ_1, τ_2) -continuous if F is lower contra- (τ_1, τ_2) -continuous at each point x of X.

Theorem 2. For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:

- (1) F is lower contra- (τ_1, τ_2) -continuous;
- (2) $F^{-}(K)$ is $\tau_{1}\tau_{2}$ -open in X for every $\sigma_{1}\sigma_{2}$ -closed set K of Y;
- (3) $F^+(V)$ is $\tau_1\tau_2$ -closed in X for every $\sigma_1\sigma_2$ -open set V of Y;
- (4) for each $x \in X$ and each $\sigma_1 \sigma_2$ -closed set K of Y such that $F(x) \cap K \neq \emptyset$, there exists a $\tau_1 \tau_2$ -open set U of X containing x such that if $y \in U$, then $F(y) \cap K \neq \emptyset$.

Proof. The proof is similar to that of Theorem 1.

Theorem 3. Let $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a multifunction. If $\tau_1 \tau_2 - Cl(F^-(B)) \subseteq F^-(\sigma_1 \sigma_2 - ker(B))$ for every subset B of Y, then F is upper contra- (τ_1, τ_2) -continuous.

Proof. Suppose that $\tau_1\tau_2$ -Cl $(F^-(B)) \subseteq F^-(\sigma_1\sigma_2$ -ker(B)) for every subset B of Y. Let V be any $\sigma_1\sigma_2$ -open set of Y. By Lemma 2, we have

$$\tau_1\tau_2\operatorname{-Cl}(F^-(V)) \subseteq F^-(\sigma_1\sigma_2\operatorname{-}ker(V)) = F^-(V)$$

and hence $F^{-}(V)$ is $\tau_1\tau_2$ -closed in X. By Theorem 1, F is upper contra- (τ_1, τ_2) -continuous.

Theorem 4. Let $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a multifunction. If $F(\tau_1\tau_2 - Cl(A)) \subseteq \sigma_1\sigma_2$ -ker(F(A)) for every subset A of X, then F is lower contra- (τ_1, τ_2) -continuous.

Proof. Let V be any $\sigma_1 \sigma_2$ -open set of Y. Then, $F(\tau_1 \tau_2 - \operatorname{Cl}(F^+(V))) \subseteq \sigma_1 \sigma_2 - ker(V)$ and hence $\tau_1 \tau_2 - \operatorname{Cl}(F^+(V)) \subseteq F^+(\sigma_1 \sigma_2 - ker(V))$. By Lemma 2, we have

$$\tau_1 \tau_2 \operatorname{-Cl}(F^+(V)) \subseteq F^+(\sigma_1 \sigma_2 \operatorname{-ker}(V)) = F^+(V)$$

and hence $F^+(V)$ is $\tau_1\tau_2$ -closed in X. By Theorem 2, F is lower contra- (τ_1, τ_2) -continuous.

Theorem 5. Let $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a multifunction. If $\tau_1 \tau_2 - Cl(F^+(B)) \subseteq F^+(\sigma_1 \sigma_2 - ker(B))$ for every subset B of Y, then F is lower contra- (τ_1, τ_2) -continuous.

Proof. Let V be any $\sigma_1 \sigma_2$ -open set of Y. Then, $\tau_1 \tau_2$ -Cl $(F^+(V)) \subseteq F^+(\sigma_1 \sigma_2 - ker(V))$ and by Lemma 2, $\tau_1 \tau_2$ -Cl $(F^+(V)) \subseteq F^+(\sigma_1 \sigma_2 - ker(V)) = F^+(V)$. This implies that $F^+(V)$ is $\tau_1 \tau_2$ -closed in X. By Theorem 2, F is lower contra- (τ_1, τ_2) -continuous. N. Viriyapong, A. Sama-Ae, C. Boonpok / Eur. J. Pure Appl. Math, 18 (2) (2025), 6008 6 of 15

Definition 3. [6] A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be upper weakly (τ_1, τ_2) -continuous if for each $x \in X$ and each $\sigma_1 \sigma_2$ -open set V of Y containing F(x), there exists a $\tau_1 \tau_2$ -open set U of X containing x such that $F(U) \subseteq \sigma_1 \sigma_2$ -Cl(V).

Theorem 6. If $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is an upper contra- (τ_1, τ_2) -continuous multifunction, then F is upper weakly (τ_1, τ_2) -continuous.

Proof. Let $x \in X$ and V be any $\sigma_1 \sigma_2$ -open set of Y containing F(x). Then, $\sigma_1 \sigma_2$ -Cl(V) is a $\sigma_1 \sigma_2$ -closed set Y containing F(x). Since F is upper contra- (τ_1, τ_2) -continuous, by Theorem 1 there exists a $\tau_1 \tau_2$ -open set U of X containing x such that $U \subseteq F^+(\sigma_1 \sigma_2$ -Cl(V)); hence $F(U) \subseteq \sigma_1 \sigma_2$ -Cl(V). This shows that F is upper weakly (τ_1, τ_2) -continuous.

The converse of Theorem 6 is not true in general as shown in the following example.

Example 1. Let $X = \{a, b, c, d\}$ with topologies $\tau_1 = \{\emptyset, \{a\}, \{a, b\}, \{a, b, c\}, X\}$ and $\tau_2 = \{\emptyset, \{a\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}, X\}$. Let $Y = \{1, 2, 3, 4\}$ with topologies

$$\sigma_1 = \{\emptyset, \{1\}, \{1, 2\}, \{1, 2, 3\}, \{1, 2, 4\}, Y\}$$

and $\sigma_2 = \{\emptyset, \{1\}, \{1, 2\}, \{1, 2, 3\}, Y\}$. A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is defined as follows: $F(a) = \{1, 2\}, F(b) = \{2\}, F(c) = \{1, 2\}$ and $F(d) = \{4\}$. Then, F is upper weakly (τ_1, τ_2) -continuous but F is not upper contra- (τ_1, τ_2) -continuous.

Definition 4. [6] A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be lower weakly (τ_1, τ_2) -continuous if for each $x \in X$ and each $\sigma_1 \sigma_2$ -open set V of Y such that $F(x) \cap V \neq \emptyset$, there exists a $\tau_1 \tau_2$ -open set U of X containing x such that $\sigma_1 \sigma_2$ - $Cl(V) \cap F(z) \neq \emptyset$ for each $z \in U$.

Theorem 7. If $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is a lower contra- (τ_1, τ_2) -continuous multifunction, then F is lower weakly (τ_1, τ_2) -continuous.

Proof. The proof is similar to that of Theorem 6.

Recall that a bitopological space (X, τ_1, τ_2) is said to be $\tau_1 \tau_2$ -connected [77] if X cannot be written as the union of two nonempty disjoint $\tau_1 \tau_2$ -open sets.

Lemma 3. [6] For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:

(1) F is upper weakly (τ_1, τ_2) -continuous;

(2)
$$F^+(V) \subseteq \tau_1 \tau_2$$
-Int $(F^+(\sigma_1 \sigma_2 - Cl(V)))$ for every $\sigma_1 \sigma_2$ -open set V of Y;

- (3) $\tau_1\tau_2$ -Cl($F^-(\sigma_1\sigma_2$ -Int(K))) $\subseteq F^-(K)$ for every $\sigma_1\sigma_2$ -closed set K of Y;
- (4) $\tau_1\tau_2$ - $Cl(F^-(\sigma_1\sigma_2$ - $Int(\sigma_1\sigma_2$ - $Cl(B)))) \subseteq F^-(\sigma_1\sigma_2$ -Cl(B)) for every subset B of Y;
- (5) $F^+(\sigma_1\sigma_2\operatorname{-Int}(B)) \subseteq \tau_1\tau_2\operatorname{-Int}(F^+(\sigma_1\sigma_2\operatorname{-Cl}(\sigma_1\sigma_2\operatorname{-Int}(B))))$ for every subset B of Y;

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- (6) $\tau_1\tau_2$ -Cl($F^-(\sigma_1\sigma_2$ -Int($\sigma_1\sigma_2$ -Cl(V)))) $\subseteq F^-(\sigma_1\sigma_2$ -Cl(V)) for every $\sigma_1\sigma_2$ -open set V of Y;
- (7) $\tau_1\tau_2$ -Cl($F^-(V)$) $\subseteq F^-(\sigma_1\sigma_2$ -Cl(V)) for every $\sigma_1\sigma_2$ -open set V of Y;
- (8) $\tau_1\tau_2$ -Cl($F^-(\sigma_1\sigma_2$ -Int(K))) $\subseteq F^-(K)$ for every (σ_1, σ_2) r-closed set K of Y.

Lemma 4. [6] For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:

- (1) F is lower weakly (τ_1, τ_2) -continuous;
- (2) $F^{-}(V) \subseteq \tau_{1}\tau_{2}$ -Int $(F^{-}(\sigma_{1}\sigma_{2}-Cl(V)))$ for every $\sigma_{1}\sigma_{2}$ -open set V of Y;
- (3) $\tau_1\tau_2$ -Cl($F^+(\sigma_1\sigma_2$ -Int(K))) $\subseteq F^+(K)$ for every $\sigma_1\sigma_2$ -closed set K of Y;
- (4) $\tau_1\tau_2$ -Cl($F^+(\sigma_1\sigma_2$ -Int($\sigma_1\sigma_2$ -Cl(B)))) $\subseteq F^+(\sigma_1\sigma_2$ -Cl(B)) for every subset B of Y;
- (5) $F^{-}(\sigma_{1}\sigma_{2}\text{-}Int(B)) \subseteq \tau_{1}\tau_{2}\text{-}Int(F^{-}(\sigma_{1}\sigma_{2}\text{-}Cl(\sigma_{1}\sigma_{2}\text{-}Int(B))))$ for every subset B of Y;
- (6) $\tau_1\tau_2$ -Cl(F⁺($\sigma_1\sigma_2$ -Int($\sigma_1\sigma_2$ -Cl(V)))) \subseteq F⁺($\sigma_1\sigma_2$ -Cl(V)) for every $\sigma_1\sigma_2$ -open set V of Y;
- (7) $\tau_1\tau_2$ -Cl($F^+(V)$) $\subseteq F^+(\sigma_1\sigma_2$ -Cl(V)) for every $\sigma_1\sigma_2$ -open set V of Y;
- (8) $\tau_1\tau_2$ -Cl(F⁺($\sigma_1\sigma_2$ -Int(K))) \subseteq F⁺(K) for every (σ_1, σ_2)r-closed set K of Y.

Theorem 8. If $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is an upper or lower contra- (τ_1, τ_2) -continuous surjective multifunction such that F(x) is $\sigma_1 \sigma_2$ -connected for each $x \in X$ and (X, τ_1, τ_2) is $\tau_1 \tau_2$ -connected, then (Y, σ_1, σ_2) is $\sigma_1 \sigma_2$ -connected.

Proof. Suppose that (Y, σ_1, σ_2) is not $\sigma_1 \sigma_2$ -connected. There exist nonempty $\sigma_1 \sigma_2$ open sets U and V of Y such that $U \cap V = \emptyset$ and $U \cup V = Y$. Since F(x) is $\sigma_1 \sigma_2$ -connected for each $x \in X$, either $F(x) \subseteq U$ or $F(x) \subseteq V$. If $x \in F^+(U \cup V)$, then $F(x) \subseteq U \cup V$ and hence $x \in F^+(U) \cup F^+(V)$. Moreover, since F is surjective, there exist x and y in X such that $F(x) \subseteq U$ and $F(y) \subseteq V$; hence $x \in F^+(U)$ and $y \in F^+(V)$. Therefore, we obtain the following:

- (1) $F^+(U) \cup F^+(V) = F^+(U \cup V) = X;$
- (2) $F^+(U) \cap F^+(V) = F^+(U \cap V) = \emptyset;$
- (3) $F^+(U) \neq \emptyset$ and $F^+(V) \neq \emptyset$.

Next, we show that $F^+(U)$ and $F^+(V)$ are $\tau_1\tau_2$ -open in X. (i) Let F be upper contra-(τ_1, τ_2)-continuous, by Theorem 6 we have F is upper weakly (τ_1, τ_2)-continuous. By Lemma 3, $F^+(V) \subseteq \tau_1\tau_2$ -Int($F^+(\sigma_1\sigma_2$ -Cl(V))) = $\tau_1\tau_2$ -Int($F^+(V)$) since V is $\sigma_1\sigma_2$ -clopen. Thus, $F^+(V) = \tau_1\tau_2$ -Int($F^+(V)$) and hence $F^+(V)$ is $\tau_1\tau_2$ -open in X. Similarly, we obtain $F^+(U)$ is $\tau_1\tau_2$ -open in X. Consequently, this shows that (X, τ_1, τ_2) is not $\tau_1\tau_2$ -connected. (*ii*) Let F be lower contra- (τ_1, τ_2) -continuous, by Theorem 7 we have F is lower weakly (τ_1, τ_2) -continuous. By Lemma 4, $\tau_1\tau_2$ -Cl $(F^+(V)) \subseteq F^+(\sigma_1\sigma_2$ -Cl $(V)) = F^+(V)$ since V is $\sigma_1\sigma_2$ -clopen. Therefore, $F^+(V) = \tau_1\tau_2$ -Cl $(F^+(V))$ and so $F^+(V)$ is $\tau_1\tau_2$ -closed in X. Thus, we have $F^+(U)$ is $\tau_1\tau_2$ -open in X. Similarly, we obtain $F^+(V)$ is $\tau_1\tau_2$ -open in X. Consequently, this shows that (X, τ_1, τ_2) is not $\tau_1\tau_2$ -connected. This completes the proof.

Recall that a bitopological space (X, τ_1, τ_2) is said to be $\tau_1 \tau_2$ -compact [77] if every cover of X by $\tau_1 \tau_2$ -open sets of X has a finite subcover.

Definition 5. [80] A bitopological space (X, τ_1, τ_2) is said to be strongly $S - \tau_1 \tau_2$ -closed if every cover of X by $\tau_1 \tau_2$ -closed sets of X has a finite subcover.

Theorem 9. Let $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a surjective multifunction and F(x) is strongly $S \cdot \sigma_1 \sigma_2$ -closed for each $x \in X$. If F is upper contra- (τ_1, τ_2) -continuous and (X, τ_1, τ_2) is $\tau_1 \tau_2$ -compact, then (Y, σ_1, σ_2) is strongly $S \cdot \sigma_1 \sigma_2$ -closed.

Proof. Suppose that (X, τ_1, τ_2) is $\tau_1 \tau_2$ -compact. Let $\{V_\gamma \mid \gamma \in \nabla\}$ be any cover of Y by $\sigma_1 \sigma_2$ -closed sets of Y. Since F(x) is strongly $S \cdot \sigma_1 \sigma_2$ -closed for each $x \in X$, there exists a finite subset $\nabla(x)$ of ∇ such that $F(x) \subseteq \cup \{V_\gamma \mid \gamma \in \nabla(x)\}$. Put $V(x) = \cup \{V_\gamma \mid \gamma \in \nabla(x)\}$. Then, V(x) is $\sigma_1 \sigma_2$ -closed in Y and $F(x) \subseteq V(x)$. Since F is upper contra- (τ_1, τ_2) -continuous, there exists a $\tau_1 \tau_2$ -open set U(x) of X containing x such that $F(U(x)) \subseteq V(x)$. The family $\{U(x) \mid x \in X\}$ is a $\tau_1 \tau_2$ -open cover of X. Since (X, τ_1, τ_2) is $\tau_1 \tau_2$ -compact, there exists a finite number of pints, say, $x_1, x_2, x_3, ..., x_n$ in X such that $X = \cup \{U(x_k) \mid x_k \in X; 1 \leq k \leq n\}$. Thus,

$$Y = F(X) = \bigcup \{ F(U(x_k)) \mid x_k \in X; 1 \le k \le n \} \subseteq \bigcup \{ V_{\gamma(x_k)} \mid x_k \in X; 1 \le k \le n \}.$$

This shows that (Y, σ_1, σ_2) is strongly S- $\sigma_1 \sigma_2$ -closed.

Definition 6. [56] A multifunction $F: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be:

- (1) upper (τ_1, τ_2) -continuous if for each $x \in X$ and each $\sigma_1 \sigma_2$ -open set V of Y such that $F(x) \subseteq V$, there exists a $\tau_1 \tau_2$ -open set U of X containing x such that $F(U) \subseteq V$;
- (2) lower (τ_1, τ_2) -continuous if for each $x \in X$ and each $\sigma_1 \sigma_2$ -open set V of Y such that $F(x) \cap V \neq \emptyset$, there exists a $\tau_1 \tau_2$ -open set U of X containing x such that $F(z) \cap V \neq \emptyset$ for each $z \in U$.

Lemma 5. [56] For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:

- (1) F is upper (τ_1, τ_2) -continuous;
- (2) $F^+(V)$ is $\tau_1\tau_2$ -open in X for every $\sigma_1\sigma_2$ -open set V of Y;
- (3) $F^{-}(K)$ is $\tau_{1}\tau_{2}$ -closed in X for every $\sigma_{1}\sigma_{2}$ -closed set K of Y;
- (4) $\tau_1 \tau_2$ -Cl($F^-(B)$) $\subseteq F^-(\sigma_1 \sigma_2$ -Cl(B)) for every subset B of Y;

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(5)
$$F^+(\sigma_1\sigma_2\operatorname{-Int}(B)) \subseteq \tau_1\tau_2\operatorname{-Int}(F^+(B))$$
 for every subset B of Y

Theorem 10. If $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is upper (τ_1, τ_2) -continuous and

$$G: (Y, \sigma_1, \sigma_2) \to (Z, \rho_1, \rho_2)$$

is upper contra- (σ_1, σ_2) -continuous, then $G \circ F : (X, \tau_1, \tau_2) \to (Z, \rho_1, \rho_2)$ is upper contra- (τ_1, τ_2) -continuous.

Proof. Let K be any $\rho_1\rho_2$ -closed set of Z. Since G is upper contra- (σ_1, σ_2) -continuous, by Theorem 1 we have $F^+(K)$ is $\sigma_1\sigma_2$ -open in Y. Since F is upper (τ_1, τ_2) -continuous, by Lemma 5 we have $(G \circ F)^+(K) = F^+(G^+(K))$ is $\tau_1\tau_2$ -open in X. Thus by Theorem 1, $G \circ F$ is upper contra- (τ_1, τ_2) -continuous.

Lemma 6. [56] For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:

- (1) F is lower (τ_1, τ_2) -continuous;
- (2) $F^{-}(V)$ is $\tau_{1}\tau_{2}$ -open in X for every $\sigma_{1}\sigma_{2}$ -open set V of Y;
- (3) $F^+(K)$ is $\tau_1\tau_2$ -closed in X for every $\sigma_1\sigma_2$ -closed set K of Y;
- (4) $\tau_1\tau_2$ -Cl(F⁺(B)) \subseteq F⁺($\sigma_1\sigma_2$ -Cl(B)) for every subset B of Y;
- (5) $F(\tau_1\tau_2 Cl(A)) \subseteq \sigma_1\sigma_2 Cl(F(A))$ for every subset A of X;
- (6) $F^{-}(\sigma_{1}\sigma_{2}\text{-Int}(B)) \subseteq \tau_{1}\tau_{2}\text{-Int}(F^{-}(B))$ for every subset B of Y.

Theorem 11. If $F: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is lower (τ_1, τ_2) -continuous and

$$G: (Y, \sigma_1, \sigma_2) \to (Z, \rho_1, \rho_2)$$

is lower contra- (σ_1, σ_2) -continuous, then $G \circ F : (X, \tau_1, \tau_2) \to (Z, \rho_1, \rho_2)$ is lower contra- (τ_1, τ_2) -continuous.

Proof. The proof is similar to that of Theorem 10.

For a multifunction $F: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, a multifunction

$$\operatorname{Cl} F_{\circledast} : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$$

is defined in [77] as follows: $\operatorname{Cl} F_{\circledast}(x) = \sigma_1 \sigma_2 \operatorname{-Cl}(F(x))$ for each $x \in X$.

Definition 7. [77] A subset A of a bitopological space (X, τ_1, τ_2) is said to be:

(1) $\tau_1\tau_2$ -paracompact if every cover of A by $\tau_1\tau_2$ -open sets of X is refined by a cover of A which consists of $\tau_1\tau_2$ -open sets of X and is $\tau_1\tau_2$ -locally finite in X;

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(2) $\tau_1\tau_2$ -regular if for each $x \in A$ and each $\tau_1\tau_2$ -open set U of X containing x, there exists a $\tau_1\tau_2$ -open set V of X such that $x \in V \subseteq \tau_1\tau_2$ - $Cl(V) \subseteq U$.

Lemma 7. [77] If $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is a multifunction such that F(x) is $\sigma_1 \sigma_2$ -regular and $\sigma_1 \sigma_2$ -paracompact for each $x \in X$, then $ClF^+_{\circledast}(V) = F^+(V)$ for each $\sigma_1 \sigma_2$ -open set V of Y.

Lemma 8. If $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is a multifunction such that F(x) is $\sigma_1 \sigma_2$ -regular and $\sigma_1 \sigma_2$ -paracompact for each $x \in X$, then $ClF^-_{\circledast}(K) = F^-(K)$ for each $\sigma_1 \sigma_2$ -closed set K of Y.

Proof. It follows from Lemma 7.

Lemma 9. [77] For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2), \ ClF^-_{\circledast}(V) = F^-(V)$ for each $\sigma_1 \sigma_2$ -open set V of Y.

Lemma 10. For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, $ClF^+_{\circledast}(K) = F^+(K)$ for each $\sigma_1 \sigma_2$ -closed set K of Y.

Proof. It follows from Lemma 9.

Theorem 12. A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is upper contra- (τ_1, τ_2) continuous if and only if $ClF_{\circledast} : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is upper contra- (τ_1, τ_2) -continuous.

Proof. Suppose that F is upper contra- (τ_1, τ_2) -continuous. Let K be any $\sigma_1 \sigma_2$ -closed set of Y. It follows from Lemma 9, Lemma 10 and Theorem 1, $\operatorname{Cl}F^+_{\circledast}(K) = F^+(K)$ is $\tau_1 \tau_2$ -open in X. Thus, $\operatorname{Cl}F_{\circledast}$ is upper contra- (τ_1, τ_2) -continuous.

Conversely, suppose that $\operatorname{Cl} F_{\circledast}$ is upper contra- (τ_1, τ_2) -continuous. Let K be any $\sigma_1 \sigma_2$ closed set of Y. By Lemma 9, Lemma 10 and Theorem 1, $F^+(K) = \operatorname{Cl} F^+_{\circledast}(K)$ is $\tau_1 \tau_2$ -open in X. Thus, F is upper contra- (τ_1, τ_2) -continuous.

Theorem 13. Let $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a multifunction such that F(x) is $\sigma_1 \sigma_2$ paracompact and $\sigma_1 \sigma_2$ -regular for each $x \in X$. Then, F is lower contra- (τ_1, τ_2) -continuous
if and only if $ClF_{\circledast} : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is lower contra- (τ_1, τ_2) -continuous.

Proof. Suppose that F is lower contra- (τ_1, τ_2) -continuous. Let K be any $\sigma_1 \sigma_2$ -closed set of Y. It follows from Lemma 7, Lemma 8 and Theorem 2 that $\operatorname{Cl} F^-_{\circledast}(K) = F^-(K)$ is $\tau_1 \tau_2$ -open in X. This shows that $\operatorname{Cl} F_{\circledast}$ is lower contra- (τ_1, τ_2) -continuous.

Conversely, suppose that $\operatorname{Cl} F_{\circledast}$ is lower contra- (τ_1, τ_2) -continuous. Let K be any $\sigma_1 \sigma_2$ closed set of Y. By Lemma 7, Lemma 8 and Theorem 2, $F^-(K) = \operatorname{Cl} F^-_{\circledast}(K)$ is $\tau_1 \tau_2$ -open in X. This shows that F is lower contra- (τ_1, τ_2) -continuous.

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